



St Robert of Newminster

Catholic School and Sixth Form College

# **Numeracy Across the Curriculum and Methods for Mathematics**

A Guide for Parents

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## **Introduction**

At St. Robert's School, we understand the important role that parents and carers can play in helping their child to develop confidence within applying numeracy and mathematics skills across all subjects and in daily life.

Numeracy skills are used in almost every subject that pupils learn at school, in some form, and are vital to helping them build a successful future beyond their school years. It is imperative that pupils leave school with numeracy skills that can help them in their daily life. Some of the most important examples of this include maintaining a good job and dealing with home-owner responsibilities, such as paying bills and finding the best deals. According to the National Numeracy Organisation, better numeracy skills are linked to improved health and wellbeing, as well as better employment and higher wages. Workplaces name numeracy skills as being one of the most common skills gaps in their employees. It is therefore crucial that every child can develop the confidence to tackle numerical problems with a positive attitude. We appreciate the important role that you may play in helping to develop this confidence and engagement to apply numeracy to their learning in a range of different contexts.

Here at St. Robert's, we want parents and carers to feel confident in helping your child with maths at home. We are fully aware that some methods that may have been used when you were taught mathematics may be different to those that your child uses. With this in mind, we have created this booklet with a range of common methods and approaches that we use to teach mathematics at St. Robert's. We hope that this can help you to support your child with applying their numeracy skills in whichever subject it may be needed.

Thank you for your ongoing support in helping your child to achieve their potential and develop the important numeracy skills and positive approach needed for their future endeavours.

**St. Robert's Mathematics Department**

## Useful Websites

[www.corbettmaths.com](http://www.corbettmaths.com)



# Corbettmaths

Welcome Videos and Worksheets Primary 5-a-day  $\vee$  More  $\vee$  Revision Cards

Welcome

**5-a-day**

**Videos**

**Worksheets**

This is a fantastic website with videos that explain the knowledge needed for each topic taught in the maths curriculum, with fully worked examples. There are also worksheets with worked answers for extra practice.

## **Worded Problems**

In maths, one of the most common areas that pupil express difficulty with is worded problems. Below is a mnemonic of a systematic approach that pupils should follow when tackling these types of questions.

**R**ead – have you read the worded question?

**U**nderstand and underline - have you understood the question and highlighted the key words or figures?

**C**hoose and calculate – choose the correct operation (+, −, x, ÷) and convert the key words into numbers or maths symbols.

**S**olve – can you solve the calculation, showing your working out?

**A**nswer – write your answer clearly, in a full sentence if needed and make sure you include the units in your answer (e.g. cm)

**C**heck – check that your answer is reasonable and if possible, substitute it back into the original question to check.

## Graphs

When working with any type of graph, it needs to be clear and fit for purpose. Below is a mnemonic to help pupils check whether their graphs are of good quality.

**S**cale – does your graph have a clear, suitable, accurate unbroken scale on both the horizontal and vertical, with even increments?

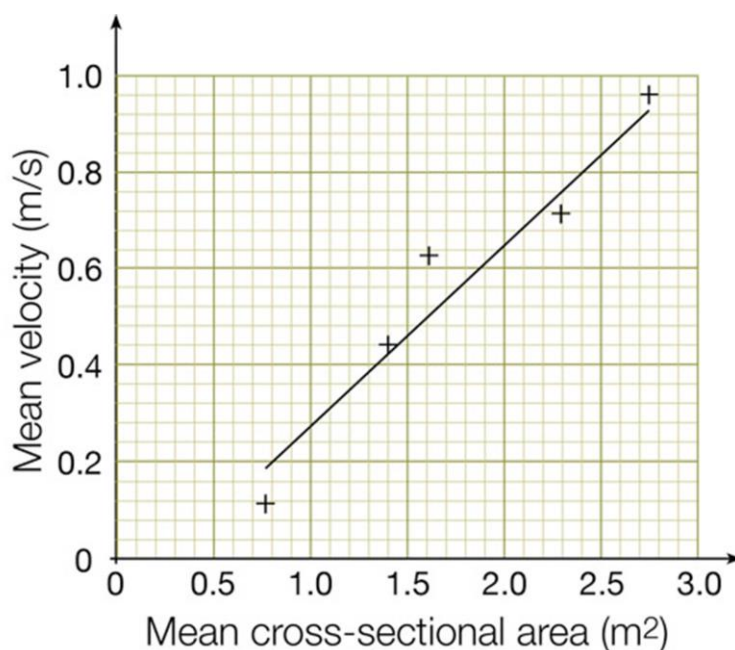
**A**xes – are the axes showing the correct information with a suitable scale (are the  $x$  and  $y$  axes the correct way around)?

**L**abel – are both axes labelled correctly, including units?

**T**itle – does the graph have a meaningful title that explains the graph?

Example of a graph used in Geography

Average flow velocity rate of a river.



## Calculations

### Addition and Subtraction

$$\begin{array}{r} 5 \quad 6 \quad 8 \\ + \quad \underline{1 \quad 4 \quad 1 \quad 7} \\ 6 \quad 1 \quad 5 \end{array}$$

We carry "on the doorstep".

$$\begin{array}{r} \quad \quad \quad \color{red}{4} \quad \color{red}{12} \quad \color{red}{1} \\ \color{blue}{/} \quad \color{blue}{/} \quad \color{blue}{/} \\ - \quad \underline{1 \quad 5 \quad 9} \\ 3 \quad 7 \quad 9 \end{array}$$

If the bottom number is bigger than the top number, then we need to "borrow".

### Multiplication

There are three different methods that students might use, depending on preference.

#### Method 1 - Column method

Work out  $23 \times 5$

$$\begin{array}{r} 2 \quad 3 \\ \underline{\quad \color{red}{1} \quad 5} \quad \times \\ 1 \quad 1 \quad 5 \end{array}$$

Work out  $23 \times 15$

$$\begin{array}{r} 2 \quad 3 \\ \underline{\color{red}{1} \quad 5} \quad \times \\ 1 \quad 1 \quad 5 \\ 2 \quad 3 \quad \color{red}{0} \quad + \\ \hline 3 \quad 4 \quad 5 \end{array}$$

### Working with decimals

When multiplying with decimals using this method, we remove the decimals and then replace them after we have done the calculation.

For example, work out  $0.23 \times 0.5$

First remove the decimals. We now have  $23 \times 5$ , which we know is 115. The original question had 3 digits after the decimal point altogether, so our answer needs 3 decimal places too.

So  $0.23 \times 0.5 = 0.115$



## Method 2 - Grid method

Work out  $23 \times 5$

x	20	3
5	100	15

$$\begin{array}{r} 100 \\ 15 \\ \hline 115 \end{array} +$$

$$23 \times 5 = 115$$

Work out  $23 \times 15$

x	20	3
10	200	30
5	100	15

$$200 + 100 + 30 + 15 = 345$$

$$\text{so } 23 \times 15 = 345$$

## Working with decimals

When multiplying with decimals using this method, we remove the decimals and then replace them after we have done the calculation.

For example, work out  $0.23 \times 0.5$

First remove the decimals. We now have  $23 \times 5$ , which we know is 115. The original question had 3 digits after the decimal point altogether, so our answer needs 3 decimal places too.

$$\text{So } 0.23 \times 0.5 = 0.115$$

### Method 3 - Lattice Method

$37 \times 56 =$

Write the numbers to be multiplied on the top and on the right.

Multiply the number on top of the box with the number to the right of the box and write the answer in the box

Add the numbers in the box diagonally and carry to the next column when necessary.

$37 \times 56 = 2072$

### Working with decimals

### Division – “The Bus Stop Method”

No remainder:

$750 \div 3$

$$\begin{array}{r} 250 \\ 3 \overline{) 750} \end{array}$$

With remainder:

$750 \div 4$

$$\begin{array}{r} 187.5 \\ 4 \overline{) 750.0} \end{array}$$

## Ordering Decimals

Order these from smallest to largest.

8.1      8.12      8.01      8.305      8.2

Line up the place value of the digits and add zeros so they all have the same number of decimal places.

			→		
8	.	1	0	0	
8	.	1	2	0	
8	.	0	1	0	
8	.	3	0	5	
8	.	2	0	0	

Work your way across the place value columns from left to right to arrange the values from smallest to largest.

Answer:              8.01      8.1      8.12      8.2      8.305

## Negative Numbers

++ → +  
-- → +  
+- → -  
-+ → -

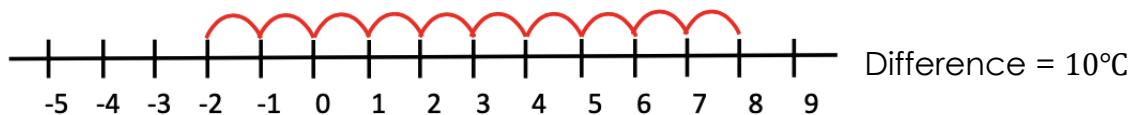
$$2 + -3 = 2 - 3 = -1$$

$$2 - -3 = 2 + 3 = 5$$

$$2 \times -3 = -6$$

$$-18 \div -6 = +3$$

Find the difference between  $-2^{\circ}\text{C}$  and  $8^{\circ}\text{C}$ . Use a number line.



## Order of Operations (BIDMAS)

The order in which calculations are carried out is referred to as BIDMAS (or also sometimes called BODMAS).

**B**rackets

**I**ndices

**D**ivision

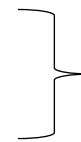
**M**ultiplication

**A**ddition

**S**ubtraction



Done in order from left to right  
(like reading a sentence).



Done in order from left to right  
(like reading a sentence).

Work out the value of  $5 \times (12 - 5) + 3^2$ .

$$5 \times (12 - 5) + 3^2$$

$$= 5 \times 7 + 3^2$$

$$= 5 \times 7 + 9$$

$$= 35 + 9$$

$$= 44$$

Note here we have addition and subtraction, which is done from left to right.

$$10 - 3 + 2$$

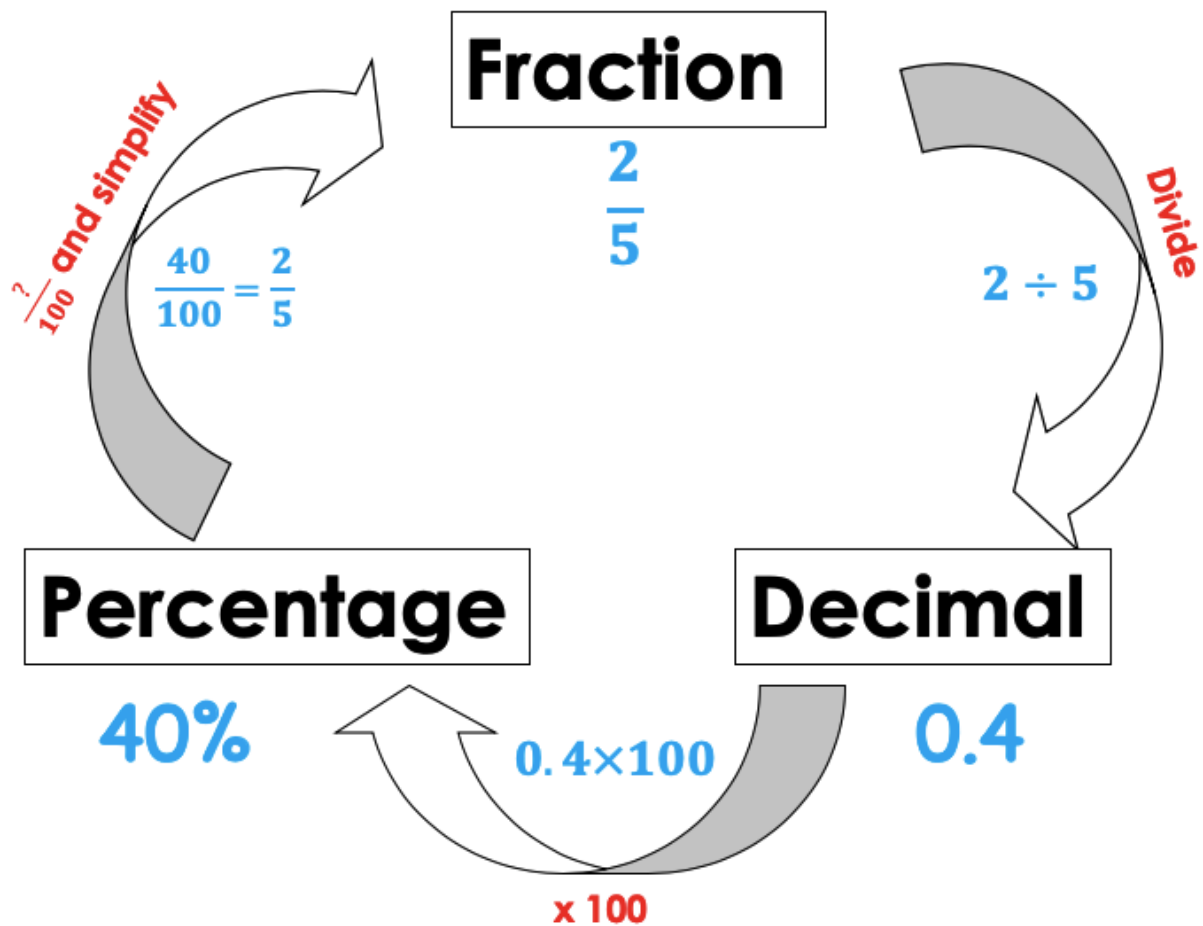
$$= 7 + 2$$

$$= 9$$

## Fractions, Decimals and Percentages

### Key conversions

<u>Fraction</u>	<u>Decimal</u>	<u>Percentage</u>
$\frac{1}{10}$	0.1	10%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{3}$	0.333...	33.333...%
$\frac{1}{2}$	0.5	50%
$\frac{2}{3}$	0.666...	66.666...%
$\frac{3}{4}$	0.75	75%



### Fraction Conversion

Convert  $\frac{3}{8}$  to a decimal and a percentage.

$$\begin{array}{r}
 0.375 \\
 8 \overline{) 3.3000}
 \end{array}$$

Convert it to a decimal first using the bus stop method.

$$0.375 \times 100 = 37.5\%$$

To convert it to a percentage, multiply by 100. The digits move two places to the left.

## Decimal Conversion

Convert 0.3 to a fraction and a percentage.

**H T U . Tenths**

0 . 3

We have three tenths, which is written as  $\frac{3}{10}$ .

To convert to a percentage, we multiply the decimal by 100.

$$0 . 3 \times 100 = 30\%$$

---

Convert 0.125 to a fraction and a percentage.

**H T U . Tenths Hundredths Thousandths**

0 . 1 2 5

We have one hundred and twenty five thousandths, which is written as

$$\frac{125}{1000} = \frac{1}{8}$$

To convert to a percentage, we multiply the decimal by 100.

$$0 . 125 \times 100 = 12.5\%$$

## Percentage Conversion

Convert 25% to a fraction and a decimal.

25% is 25 “per cent”, which is written as  $\frac{25}{100}$ . This simplifies to  $\frac{1}{4}$ .

To convert from a percentage to a decimal, divide by 100.

$$25 \div 100 = 0.25$$

## Percentage of an Amount

<b>Percent</b>	<b>How to work it out</b>
50%	Halve the quantity
25%	Quarter the quantity (halve then halve again)
10%	Tenth (divide by 10)
5%	Find 10% then halve it
1%	Hundredth (divide by 100)

### Non-Calculator Method

To find 63% of 200 without a calculator:

$$50\% \text{ of } 200 = 100$$

$$10\% \text{ of } 200 = 20$$

$$1\% \text{ of } 200 = 2$$

$$3\% \text{ of } 200 = 6$$

$$\begin{aligned} \text{So } 63\% &= 50\% + 10\% + 3\% \\ &= 100 + 20 + 6 \\ &= 126 \end{aligned}$$

To increase 200 by 63%, we find 63% and add it on to the 200.

$$200 + 126 = 326$$

### Calculator Method

To find 63% of 200 with a calculator:

We know  $63\% = 0.63$  as a decimal.

$$200 \times 0.63 = 126$$

To increase 200 by 63%:

We start with 100% of the original value. We add on 63%, so we end up with 163%.

163% as a decimal is 1.63

$$200 \times 1.63 = 326$$



## Expressing as a Percentage

Express 5 as a percentage of 20.

$$\frac{5}{20} = 0.25 = 25\%$$

Alternatively, we could have done the following:

$$\frac{5}{20} = \frac{25}{100} = 25\%$$

## Percentage Change (Profit and Loss)

$$\text{Percentage Change} = \frac{\text{difference}}{\text{original value}} \times 100$$

A coat originally cost £30 and now costs £25. Find the percentage reduction in the sale.

$$\text{Percentage change} = \frac{\pounds 40 - \pounds 25}{\pounds 40} \times 100$$

$$= \frac{15}{40} \times 100$$

$$= 37.5\%$$

A car that cost £3500 to buy is sold for £5000. Work out the percentage profit.

$$\text{Percentage change} = \frac{\pounds 5000 - \pounds 3500}{\pounds 3500} \times 100$$

$$= \frac{1500}{3500} \times 100$$

$$= 42.9\%$$

## Working with Fractions

### Addition and Subtraction

#### Example 1 – Same denominators

Example Find  $\frac{3}{7} + \frac{2}{7}$

These have the same denominators so we just add their numerators – we don't add or subtract the denominators

$$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$$

#### Example 2 – Different denominators

$$\begin{array}{r} \frac{2}{3} \\ \times 8 \\ \hline 16 \end{array} - \begin{array}{r} \frac{3}{8} \\ \times 3 \\ \hline 9 \end{array} = \frac{16}{24} - \frac{9}{24}$$

The denominators are different. To make them equal, choose the smallest common multiple of both 3 and 8, which is 24.

$$= \frac{7}{24}$$

Another method is the “butterfly” or “kiss and a smile” method. Multiply the denominators and cross multiply the wings.

Butterfly Method for Fractions (Addition & Subtraction)

$$\begin{array}{r} 6 \quad + \quad 35 = 41 \\ \frac{3}{7} + \frac{5}{2} \\ \hline 14 \end{array}$$

$$\frac{3}{7} + \frac{5}{2} = \frac{41}{14}$$

## Multiplication

Multiply across the top and multiply across the bottom.

$$\frac{2}{3} \times \frac{5}{9} = \frac{10}{27}$$

## Division

We use the mnemonic KFC.

**K** Keep the first fraction as it is

**F**lip the second fraction upside down

**C**hange the  $\div$  sign to a  $\times$  sign

$$\frac{11}{12} \div \frac{3}{7} = \frac{11}{12} \times \frac{7}{3} = \frac{11 \times 7}{12 \times 3} = \frac{77}{36}$$

1) Keep the first fraction the same

2) Flip the 2<sup>nd</sup> fraction upside down

3) Change the  $\div$  sign to a  $\times$  sign

## Mixed Number to Improper Fraction

$$2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{11}{4}$$

## Improper Fraction to Mixed Number

Convert  $\frac{11}{4}$  to a mixed number.

Method: How many times does 4 divide into 11?  
4 divides into 11 exactly 2 times with a remainder of 3.

$$\text{So } \frac{11}{4} = 2\frac{3}{4}$$

### Fraction of an Amount

Find  $\frac{2}{3}$  of 15.

Find one third first by dividing 15 by 3, which equals 5.

We want two thirds, so multiply by 2.

$$5 \times 2 = 10.$$

Hint: "Divide by the bottom and multiply by the top".

### Simplifying Fractions

Fully simplify  $\frac{36}{48}$

$$\begin{array}{ccccc} & \div 3 & & \div 2 & & \div 2 & & \\ & \curvearrowright & & \curvearrowright & & \curvearrowright & & \\ \frac{36}{48} & = & \frac{12}{16} & = & \frac{6}{8} & = & \frac{3}{4} & \\ & \curvearrowleft & & \curvearrowleft & & \curvearrowleft & & \\ & \div 3 & & \div 2 & & \div 2 & & \end{array}$$

### Expressing an Amount as a Fraction

Express 6 days as a fraction of 2 weeks.

First, ensure that the units are the same. 2 weeks is equivalent to 14 days.

$$\frac{6 \text{ days}}{2 \text{ weeks}} = \frac{6 \text{ days}}{14 \text{ days}} = \frac{6}{14} = \frac{3}{7}$$

## Rounding

Pupils are taught the rules:

“5 or more” rounds the digit up to the next value.

“4 or less” makes the digit stay the same.

### Nearest 10, 100, 1000, whole number (integer)

#### Example 1

Round 17 to the nearest **ten**.

Tens	Units
<u>1</u>	7

Underline the place value column that you are interested in rounding the number to. Put an arrow to the digit to the right of it.

The 7 is “5 or more” so it tells the 1 in the tens to move up to a 2, so it is now worth twenty. Fill the units with a zero as a place holder.

Answer: 17 rounded to the nearest 10 is 20.

#### Example 2

Round 230 to the nearest **hundred**.

Hundreds	Tens	Units
<u>2</u>	3	0

The 3 is “4 or less” so it tells the 2 to “stay the same”. Fill the columns to the right of the 2 with zeros as place holders.

230 rounded to the nearest 100 is 200.

#### Example 3

Round 56890 to the nearest **thousand**.

Ten thousands	Thousands	Hundreds	Tens	Units
5	<u>6</u>	8	9	0

The 8 is “5 or more” so it tells the 6 to move up to a 7”. Fill the columns to the right of the 6 with zeros as place holders. The 5 remains untouched.

56890 rounded to the nearest 1000 is 57000.

### **Example 4**

Round 3.1 to the nearest **whole number**.

Units    Tenths  
3 .    1  
          ↑

The 1 is "4 or less" so it tells the 3 to stay the same.

3.1 rounded to the nearest whole number is 3.

### **Nearest Decimal Place**

You start counting decimal places from the first digit after the decimal point.

	1 <sup>st</sup> d.p.	2 <sup>nd</sup> d.p.	3 <sup>rd</sup> d.p.	4 <sup>th</sup> d.p.	5 <sup>th</sup> d.p.
9 ●	1	2	3	4	5

### **Example 1**

Round 3.15 to the nearest **one decimal place (nearest tenth)**.

Units    Tenths    Hundredths  
3 .    1    5  
          ↑

The 5 is "5 or more" so it tells the 1 to move up to a 2". The 3 remains untouched.

3.15 rounded to one decimal place is 3.2

### **Example 2**

Round 3.152 to the nearest **two decimal places (nearest hundredth)**.

Units    Tenths    Hundredths    Thousandths  
3 .    1    5    2  
                                  ↑

The 2 is "4 or less" so it tells the 5 to stay the same. The 3 and the 1 remain untouched.

3.152 rounded to two decimal places is 3.15

## Significant Figures

The first significant figure (s.f.) is the first non-zero digit. The 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup> etc s.f. can be a 0. You start counting from the 1<sup>st</sup> significant figure.

### Example 1

Round 5840 to **one significant figure**.

5 8 4 0  
— ↑

The first non-zero digit from the left is the 5. The 8 tells the 5 to move up to a 6. Replace everything else with zeros. It rounds to 6000.

### Example 2

Round 0.00506 to **two significant figures**.

0 . 0 0 5 0 6  
— ↑

The first non-zero digit from the left is the 5, which is the first significant figure. The second significant figure is the 0. The 6 tells the 0 to move up to a 1. It rounds to 0.0051

## Estimation

Estimation can be used to approximate calculations or to check answers. We usually round each value to one significant figure before completing the calculation.

### Example

Estimate  $473 \times 56$ .

473 rounded to one significant figure is 500.

56 rounded to one significant figure is 60.

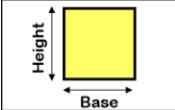
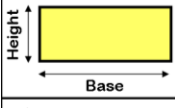
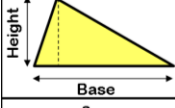
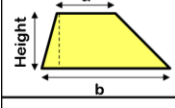
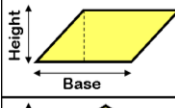
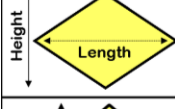

So the estimated calculation is  $500 \times 60 = 30,000$

## Area and Perimeter

Area is the square units needed to “fill” the inside of the shape.



The method for working out area depends on the shape.

Shape	Name	Formula for Area
	<b>Square</b>	Base x Height
	<b>Rectangle</b>	Base x Height
	<b>Triangle</b>	Base x Perpendicular Height ÷ 2
	<b>Trapezium</b>	$\frac{(a + b) \times \text{height}}{2}$
	<b>Parallelogram</b>	Base x Perpendicular Height
	<b>Rhombus</b>	Length x Height ÷ 2
	<b>Kite</b>	Length x Height ÷ 2

Perimeter is the distance around the outside of the shape. We add all of the side lengths together.





## Metric and Imperial Conversion

### LENGTH

#### *Metric*

1 centimetre (cm) = 10 millimetres (mm)

1 metre (m) = 100 cm

1 kilometre = 1000m

#### *Imperial*

1 foot = 12 inches

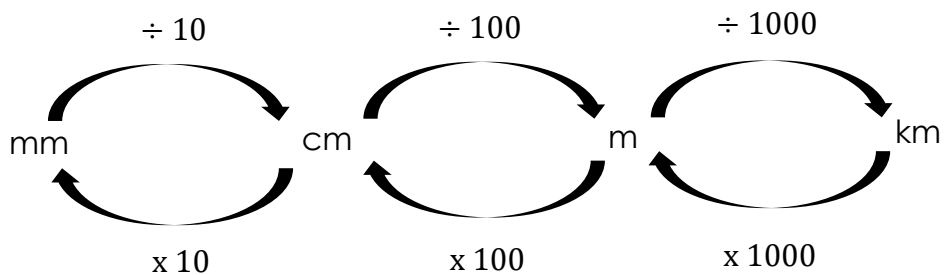
1 yard = 3 feet

1 mile = 1760 yards

#### *Metric/Imperial*

1 inch  $\approx$  2.54cm

5 miles  $\approx$  8 km



### MASS

#### *Metric*

1 gram (g) = 1000 milligrams (mg)

1 kilograms (kg) = 1000g

1 tonne = 1000 kilograms

#### *Imperial*

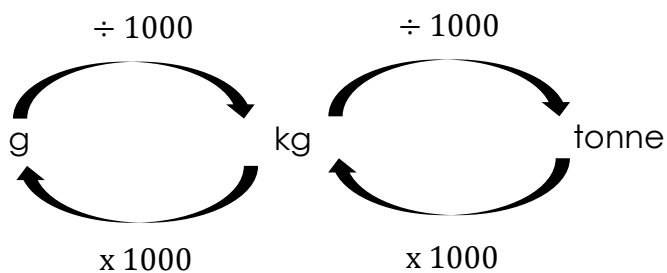
1 pound (lb) = 16 ounces (oz)

1 stone = 14 lb

1 ton = 2240 lb

#### *Metric/Imperial*

1kg  $\approx$  2.2 lb



## CAPACITY

### Metric

1 litre = 1000 millilitre (ml)

1 litre = 100 centilitres (cl)

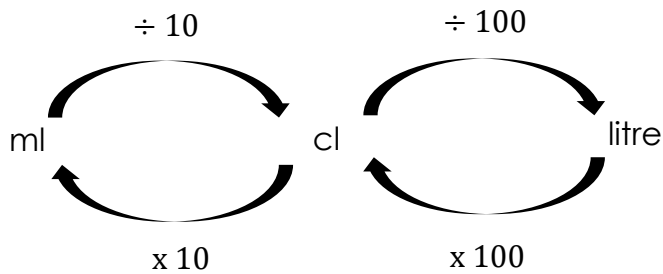
1 centilitre = 10 millilitres

### Imperial

1 gallon = 4.5 pints

### Metric/Imperial

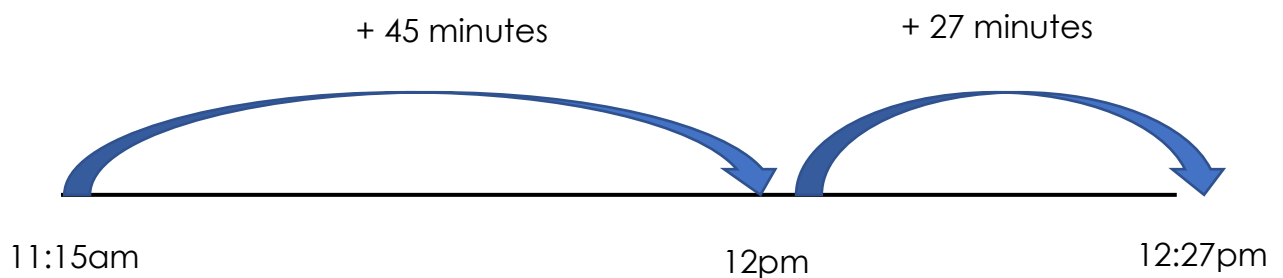
1 litre = 1.75 pints



## Time

If I leave the house at 11:15am and arrive at my destination at 12:27pm, how long did the journey take me?

Use a timeline:



Time taken =  $45 + 27 = 72$  minutes (or 1 hour and 12 minutes).

## Algebra

### Key Vocabulary

Constant	Something that does not vary, e.g. in $3x + 2$ , the constant is 2 (it never changes value).
Coefficient	The number attached to a variable, e.g. $5x$ . The coefficient is the 5 and the variable (which can take any value) is $x$ .
Expand	Multiply across all terms in the bracket to remove the brackets. E.g. $5(x + 2)$ would expand to $5x + 10$ .
Expression	A collection of letters and numbers to express a quantity. An expression does not contain an equals sign. e.g. $3x + 2$ .
Equation	An expression that is equal to something, which can be solved to find the value. e.g. the equation $3x + 2 = 14$ can be solved to give a solution of $x = 4$ .
Factorise	The opposite of expanding a bracket. $5x + 10$ factorises to $5(x + 2)$ . We took out the common <i>factor</i> of both terms, which was 5.
Formula	Similar to an equation but it has more than one variable. It is a rule relating one variable to at least one other variable. e.g. the formula for the area of a triangle is $A = \frac{b \times h}{2}$ .
Identity	This looks similar to an equation, but it is where the left-hand side and right-hand side are always true and equal, no matter what the value of the variables are. “Identical to” is shown by the symbol $\equiv$ . E.g. $x + 2 \equiv 2 + x$
Variable	Something that can take any value and is represented by a letter, usually $x$ .

## Collecting Like Terms

Simplify  $3x + 2y + 7 - x + 6y - 2$ .

$$3x + 2y + 7 - x + 6y - 2$$

Students are usually taught to use the boxes and circles or underline method to highlight which terms are alike (i.e. which have  $x$ , which have  $y$  and which may be just a constant (a number by itself)).

$$\boxed{3x} + \boxed{2y} + \underline{7} - \boxed{x} + \boxed{6y} - \underline{2}$$

$$= 2x + 8y + 5$$

A common mistake here is to put  $-5$  because pupils have seen  $+7$  and  $-2$  and think "a negative and a positive make a negative". However, if we use a number line to work out  $+7-2$ , the answer is (positive)  $5$ .

## Expanding Brackets

Most pupils will use the "Santa's Hat" method. We multiply the term outside the bracket by everything inside the bracket.

Expand  $5(3x - 1)$ .

$$5(3x - 1)$$

$$5 \times 3x = 15x$$

$$5 \times -1 = -5$$

$$5(3x - 1) = 15x - 5$$

Some pupils may use a grid method instead.

$x$	$3x$	$-1$
$5$	$15x$	$-5$

$$5(3x - 1) = 15x - 5$$

### Factorising Linear Expressions

This is the inverse of expanding brackets. Here we want to put the brackets back in. Factorise  $15x - 5$ .

Step 1: Find the highest common factor of 15 and 5, which is 5. So we now put the 5 outside the bracket.

$$5(\dots\dots\dots)$$

Step 2: Looking at the first term, what do we multiply 5 by to get the  $15x$ ?

Answer:  $3x$

Looking at the second term, what do we multiply 5 by to get  $-5$ ? Answer:  $-1$ .

$$\text{So } 15x - 5 = 5(3x - 1)$$

Some pupils may use a grid method instead and work backwards to fill in what would go in the first row and column (shown in red). The first step would still be to find the highest common factor of 15 and 5, which is 5 and the process for step 2 also follows the same method as above.

x (multiply)	$3x$	$-1$
$5$	$15x$	$-5$

$$15x - 5 = 5(3x - 1).$$

Below is an example where both a number and letter are common.

Factorise  $15x^2 - 10x$ .

Step 1: Find the highest common factor of 15 and 10, which is 5. So we now put the 5 outside the bracket. There is also a letter  $x$  common so we can also put  $x$  outside the bracket.

$$5x(\dots\dots\dots)$$

Step 2: Looking at the first term, what do we multiply  $5x$  by to get the  $15x^2$ ?

Answer:  $3x$

Looking at the second term, what do we multiply  $5x$  by to get  $-10x$ ?

Answer:  $-2$ .

$$\text{So } 15x^2 - 10x = 5x(3x - 2).$$

Alternatively, using the grid method, following step one above:

x (multiply)	$3x$	$-2$
$5x$	$15x^2$	$-10x$

## Solving Equations

When solving an equation, we are aiming to find which value of the letter makes it "true". We use the inverse operations balance method, which means we want to undo what has happened to the letter to be able to work out what it is worth. Pupils know "whatever we do to the left, we have to do to the right".

$$3x + 1 = 13$$

$$-1 \quad -1$$

$$3x = 12$$

$$\div 3 \quad \div 3$$

$$x = 4$$

To get rid of the +1, we subtract one from both sides of the equation. Then to get rid of the multiply by 3, we divide both sides by 3. We show the steps underneath in red as our method.

We can check our answer is correct by substituting the 4 back into the original question. Does  $3 \times 4 + 1 = 13$ ? The answer is yes, so my answer is correct.

We can also solve when there is a letter on both sides. Our first aim is to "get rid" of the smallest value of  $x$ . We get rid of the  $2x$  by subtracting  $2x$  from both sides.

$$6x - 5 = 2x + 23$$

$$-2x \quad -2x$$

$$4x - 5 = 23$$

$$+5 \quad +5$$

$$4x = 28$$

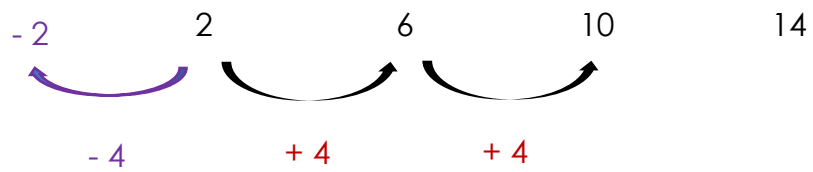
$$\div 4 \quad \div 4$$

$$x = 7$$

Again, we can check our answer by substituting the 7 back into the original question.

## Sequences

Find the  $n$ th term of the following linear sequence: 2, 6, 10, 14.



Find the difference between each term. This goes in front of  $n$ .  $4n$   
Jump back from the first term by this amount. This goes on the end of the  $n$ th term expression.

$$n\text{th term} = 4n - 2$$

## Ratio and Proportion

### Unitary Method

This method is used to find the “unit price” or unitary cost of an item to help work out any required amount.

A bag of 5 oranges costs £1.75. How much would 8 oranges cost at the same price each?

$$\begin{array}{ccc} & 5 \text{ oranges} = \text{£}1.75 & \\ \div 5 \curvearrowright & & \curvearrowleft \div 5 \\ & 1 \text{ orange} = \text{£}0.35 & \\ \times 8 \curvearrowright & & \curvearrowleft \times 8 \\ & 8 \text{ oranges} = \text{£}2.80 & \end{array}$$

### Simplifying Ratio

Simplifying a ratio is similar to simplifying a fraction. We keep dividing by a common factor. Note that we don't always divide by 2 (pupils often assume this) and we never have decimals in a ratio as a general rule.

$$\begin{array}{ccc} & 150:200 & \\ \div 2 \curvearrowright & & \curvearrowleft \div 2 \\ & 75:100 & \\ \div 5 \curvearrowright & & \curvearrowleft \div 5 \\ & 15:20 & \\ \div 5 \curvearrowright & & \curvearrowleft \div 5 \\ & 3:4 & \end{array}$$

### Ratios and Fractions

The ratio of apples to bananas is 2:3.

There are  $2 + 3 = 5$  parts in the total ratio. So  $\frac{2}{5}$  are apples.

Therefore  $\frac{3}{5}$  of them are bananas.

The fraction of sweets in a bag that are red is  $\frac{1}{4}$  and the rest are yellow. Write down the ratio of red to yellow. It helps to draw a picture here.



Using the visual, it is clearer that the ratio of red to yellow is 1:3.



## Ratio problems – “Ratio Reality and the Bar Model”

To solve ratio problems, pupils may use a range of approaches. Some methods include using bar models or “ratio-reality” tables.

Share £50 in the ratio 2:3.

### Method 1: Ratio Reality Tables

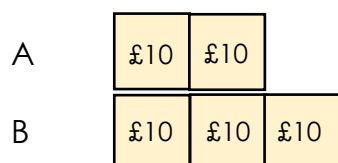
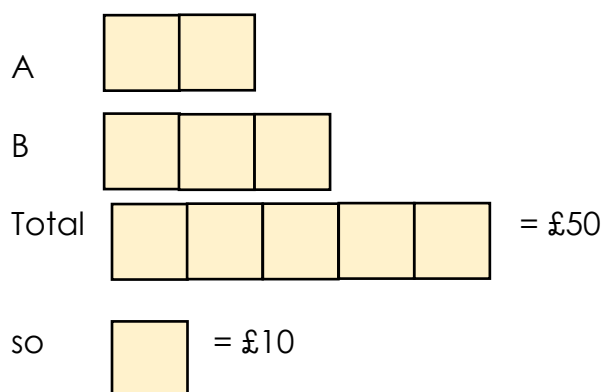
	A	B	Total
<b>Ratio</b> (what are the numbers in the ratio?)	2	3	$2 + 3 = 5$
<b>Reality</b> (in real life, what would they get?)	$2 \times 10 = \text{£}20$	$3 \times 10 = \text{£}30$	£50

$\times 10$   
This is the “one part” of the ratio.

Answer: Person A gets £20 and person B gets £30.

### Method 2: The Bar Model

Share £50 in the ratio 2:3.



So A gets £20 and B gets £30.

It may be where we are told how much one person has, rather than the total.

A and B share money in the ratio 2:3. Person A gets £40. How much did they share in total?

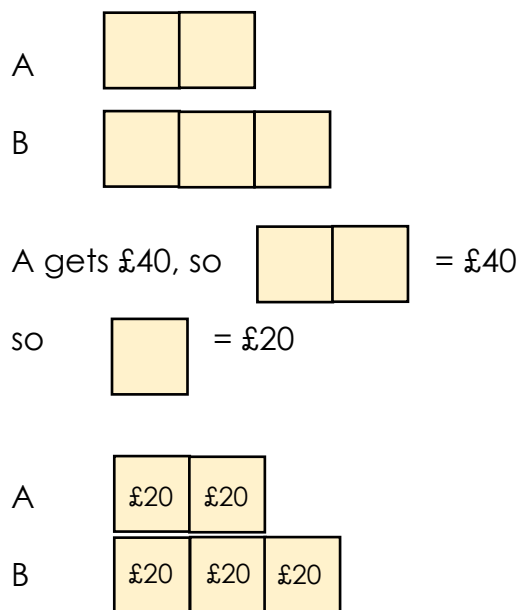
**Method 1: Ratio Reality Tables**

	A	B	Total
<b>Ratio</b> (what are the numbers in the ratio?)	2	3	$2 + 3 = 5$
<b>Reality</b> (in real life, what would they get?)	£40	$3 \times 20 = \text{£}60$	Total = £40 + £60 = £100

Total = £100

**Method 2: The Bar Model**

A and B share money in the ratio 2:3. Person A gets £40. How much did they share in total?



So A gets £40 and B gets £60, so altogether they share £100.

It may be where we are told the difference, rather than the total. This may appear in the question as “Person A gets **more** or **less** than person B, for example. In this case, we change the “total” column to a “difference” column.

A and B share money in the ratio 2:3. Person A gets £20 less than Person B. How much did Person B get?

### Method 1: Ratio Reality Tables

	A	B	Difference
<b>Ratio</b> (what are the numbers in the ratio?)	2	3	$3 - 2 = 1$
<b>Reality</b> (in real life, what would they get?)	$2 \times 20 = \text{£}40$	$3 \times 20 = \text{£}60$	$\text{£}20$

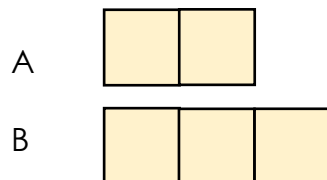
$\times 20$   
This is the “one part” of the ratio.


Answer: Person B gets £60.

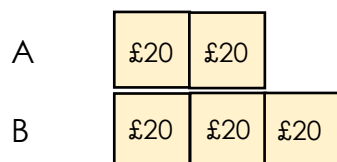
Check: is the difference between A and B actually £20?  
 $\text{£}60 - \text{£}40 = \text{£}20$  so yes, we are correct.

### Method 2: The Bar Model

A and B share money in the ratio 2:3. Person A gets £20 less than Person B. How much did Person B get?



A gets one square **less** than B (the difference between A and B is one square), so  = £20



So Person B gets £60.

Check: is the difference between A and B actually £20?  
 $\text{£}60 - \text{£}40 = \text{£}20$  so yes, we are correct.

## **Key Terminology and Command Words**

For algebraic vocabulary, see the Algebra section.

Acute	An angle that measures less than 90 degrees.
Area	The square units that fit “inside” a shape.
BIDMAS	The order in which a calculation is carried out.
Circumference	The distance around the circle.
Construct	Draw accurately, usually using compasses or a protractor.
Cube	Multiply by itself three times.
Decrease	To lower in value.
Denominator	The bottom part of a fraction.
Diameter	The distance across a circle that passes through the centre.
Difference	Subtract one number from another “how many jumps between...”
Estimate	Round all values to one significant figure before calculation.
Even	A number that is divisible exactly by 2.
Factor	A number that divides exactly into another number.
HCF	The highest common factor of two or more numbers.
Horizontal	Easily described as “side to side”.
Increase	To go up in value.
Indices	Referring to powers. E.g. $3^2$ has an index (or power) of 2.
Integer	A whole number.
Inverse	The opposite of something, e.g. the inverse of multiply is divide.
Justify	Give a reason and show mathematically why something is true.
LCM	The lowest common multiple (times tables) of two or more numbers.
Mean	Add all of the numbers up and divide by how many numbers there are.

Median	The middle value once the numbers have been put in order.
Mode	The most common item or value in a list.
Multiple	The times tables of a number.
Numerator	The top part of a fraction.
Obtuse	An angle measuring more than 90 degrees but less than 180 degrees.
Odd	A number that isn't divisible by 2.
Operation	+, -, x or ÷
Parallel	Two lines that will never meet.
Perimeter	The distance around the outside of a shape.
Perpendicular	At a right angle to.
Polygon	A closed sided shape with straight edges.
Power	Also known as the "index". E.g. $3^2$ is read as 3 raised to the power of 2.
Prime	A number that has exactly two factors; one and itself.
Product	To multiply. E.g. the product of 5 and 8 is $5 \times 8 = 40$ .
Radius	The distance from the centre of a circle to the edge.
Range	The highest value minus the lowest value.
Reflex	An angle bigger than 180 degrees.
Regular	Refers to a shape where all angles are equal, and all lengths are equal.
Root	e.g. square root. $\sqrt{4} = 2$
Show that...	When given an answer, show how that answer is achieved with method.
Sig fig	See section "Rounding to Significant Figures" above.
Square	To multiply something by itself. e.g. $5^2 = 5 \times 5 = 25$ .
Sum	Add together, e.g. the sum of 5 and 8 is $5 + 8 = 13$ .
Vertical	Easily described as "top to bottom".